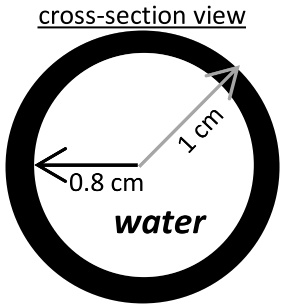
AESB2320, 2017-18

Part 2 Examination - 13 April

Write your solutions *on your answer sheet,* not here. In all cases ***show your work***.
To avoid any possible confusion,
state the equation numbers and figure numbers of equations and figures you use
along with the text you are using (BSL1, BSL2 or BSLK).
Beware of unnecessary information in the problem statement.

I have noticed that when one runs hot water at a moderate rate through the faucet at the sink by the third-floor coffee machine, it takes about 4 seconds for the *outside* of the spout to get very hot to the touch. Analyze this process in the following two steps.

1. First, heat transfer between the hot water and the faucet tube. The spout is a cylindrical tube, 30 cm long, with an inner diameter of about 1.6 cm and an outer diameter of about 2 cm. Suppose water flows through the faucet at a rate of10^{-5} m^3/s (i.e., 100 ml in 10 s). What is the heat-transfer coefficient h between the water and the pipe wall? For this part, assume for simplicity that the wall temperature is constant.

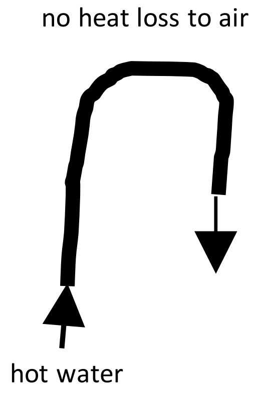


<u>Properties of water</u>

μ = 0.001 Pa s   k = 35 W/(m K) C_p = 130 J/(kg K) ρ = 1,000 kg/m^3

(30 points)

2. Second, the heating of the metal faucet pipe by the water. For the this problem, don't plug numbers in. Just assume the pipe is has length L, inner radius R_1 and outer radius R_2, and the heat-transfer coefficient on the inner surface is h. The pipe wall has density ρ, heat capacity per unit mass C_p, and thermal conductivity k. The pipe wall is initially at temperature T_0, and starting at time t = 0 water of temperature T_1 starts to flow through it. To analyze the heating of the spout by the hot water flowing in contact with it, make the following simplifying assumptions:



   a. Heat is transferred to the tube from the hot water, but there is no heat loss to air around the outside of the tube (i.e., outer surface perfectly insulated).
   b. The tube is at uniform temperature at all times as it heats.
   c. Assume the water remains at temperature T_1 all along the inner surface of the tube.

   With these assumptions, perform a macroscopic energy balance to derive a differential equation for the temperature of the pipe wall with time. Integrate this equation and obtain an algebraic equation for the temperature of the pipe wall as a function of time, given the parameters specified above.

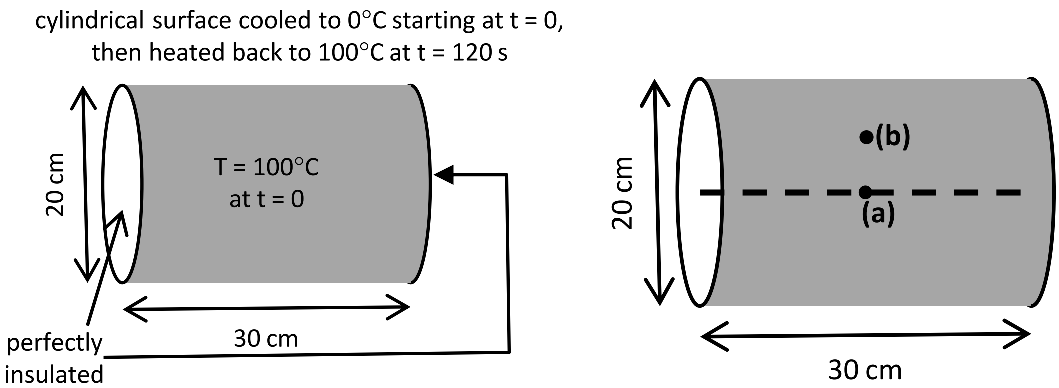
(30 points)

3. A cylinder of lead is 20 cm in diameter and 30 cm long. Both flat ends are perfectly

insulated. The cylinder is at a uniform temperature of 100°C at time t = 0. Starting at time t = 0, the cylindrical surface is suddenly cooled to 0°C and maintained at that temperature. Then, at t = 120 s, the cylindrical surface is suddenly heated back to 100°C and maintained at that temperature.

a. Consider a point (a) along the central axis of the cylinder, halfway between the flat edges. What is its temperature at a time t = 160 s, i.e. 40 s after the second change?
b. Consider a point halfway between the flat edges, but halfway to the cylindrical edge (b). What is its temperature at a time t = 160 s, i.e. 40 s after the second change?

(30 points)



<u>properties of lead</u>

$k = 34.6$ W/(m K) C_p=125.7 J/(kg K) $\rho = 11340$ kg/m^3

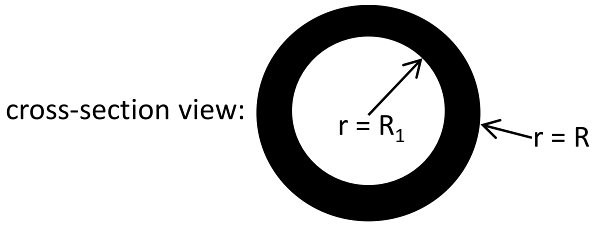
4. An electrical wire in the shape of a solid annulus extends from $r = R_1$ to $r = R$ in the radial direction. Within the solid, heat is released at a rate per unit volume S. On the inner surface of the wire, temperature is fixed at T_1; the outer surface is surrounded by air at temperature T_a, with a heat-transfer coefficient h governing heat transfer between the wire and the air. There is no temperature variation along the "z" direction, i.e. the direction along the length of the wire. The wire is at steady state.

   You are not asked to solve for T(r) here. Simply answer this question:

   You may notice that this problem is similar to that in BSL1 Section 9.2. What is the *last* equation in that derivation that can be applied directly to this problem? Write that equation number on your answer sheet, and briefly justify your answer. The relevant pages of BSL1 are provided at the end of this exam.

   Note: although the geometry looks similar, this problem has nothing to do with problems 1 or 2.

(10 points)



dependence of either the thermal or electrical conductivity need be considered. The surface of the wire is maintained at temperature T_0. We now show how one can determine the radial temperature distribution within the heated wire.

For the energy balance we select as the system a cylindrical shell of thickness Δr and length L. (See Fig. 9.2–1.) The various contributions to the energy balance are

rate of thermal energy in across cylindrical surface at r	$(2\pi rL)(q_r\|_r)$	(9.2–2)
rate of thermal energy out across cylindrical surface at $r + \Delta r$	$(2\pi(r + \Delta r)L)(q_r\|_{r+\Delta r})$	(9.2–3)
rate of production of thermal energy by electrical dissipation	$(2\pi r\,\Delta rL)S_e$	(9.2–4)

The notation q_r means "flux of energy in the r-direction," and $|_r$ means "evaluated at r." Note that we take "in" and "out" to be in the positive r-direction.

We now substitute these three expressions into Eq. 9.1–1. Division by $2\pi L\,\Delta r$ and taking the limit as Δr goes to zero gives

$$\left\{\lim_{\Delta r\to 0}\frac{(rq_r)|_{r+\Delta r} - (rq_r)|_r}{\Delta r}\right\} = S_e r \tag{9.2–5}$$

The expression within braces is just the first derivative of rq_r with respect to r, so that Eq. 9.2–5 becomes

$$\frac{d}{dr}(rq_r) = S_e r \tag{9.2–6}$$

This is a first-order ordinary differential equation for the energy flux, which may be integrated to give

$$q_r = \frac{S_e r}{2} + \frac{C_1}{r} \tag{9.2–7}$$

The integration constant C_1 must be zero because of the boundary condition

B.C. 1: at $r = 0$ q_r is not infinite (9.2–8)

Hence the final expression for the energy flux distribution is

$$\boxed{q_r = \frac{S_e r}{2}} \tag{9.2–9}$$

This states that the heat flux increases linearly with r.

We now substitute Fourier's law (see Eq. 8.1–2) in the form $q_r = -k(dT/dr)$ into Eq. 9.2–9 to obtain

$$-k\frac{dT}{dr} = \frac{S_e r}{2} \tag{9.2–10}$$

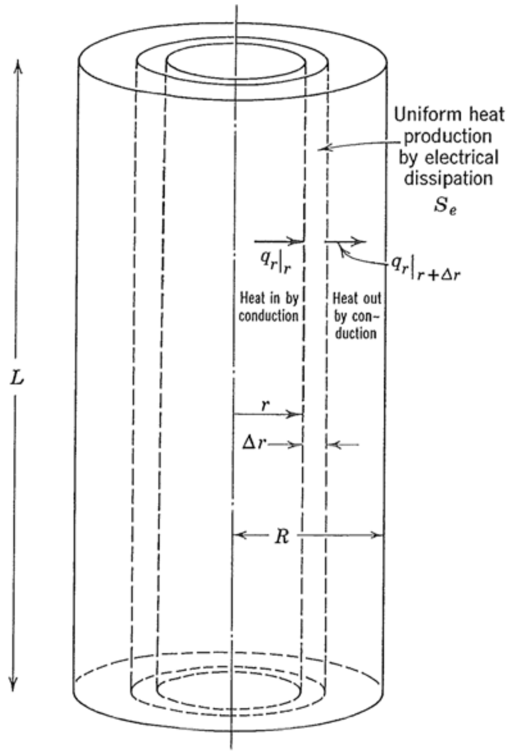


Fig. 9.2–1. Cylindrical shell over which energy balance is made in order to get temperature distribution in an electrically heated wire.

When k is assumed to be constant, this first-order differential equation may be integrated to give

$$T = -\frac{S_e r^2}{4k} + C_2 \tag{9.2–11}$$

The integration constant C_2 is determined from

B.C. 2: at $r = R$ $T = T_0$ (9.2–12)

Hence C_2 is found to be $T_0 + (S_e R^2/4k)$ and Eq. 9.2–11 becomes

$$\boxed{T - T_0 = \frac{S_e R^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right]} \tag{9.2–13}$$